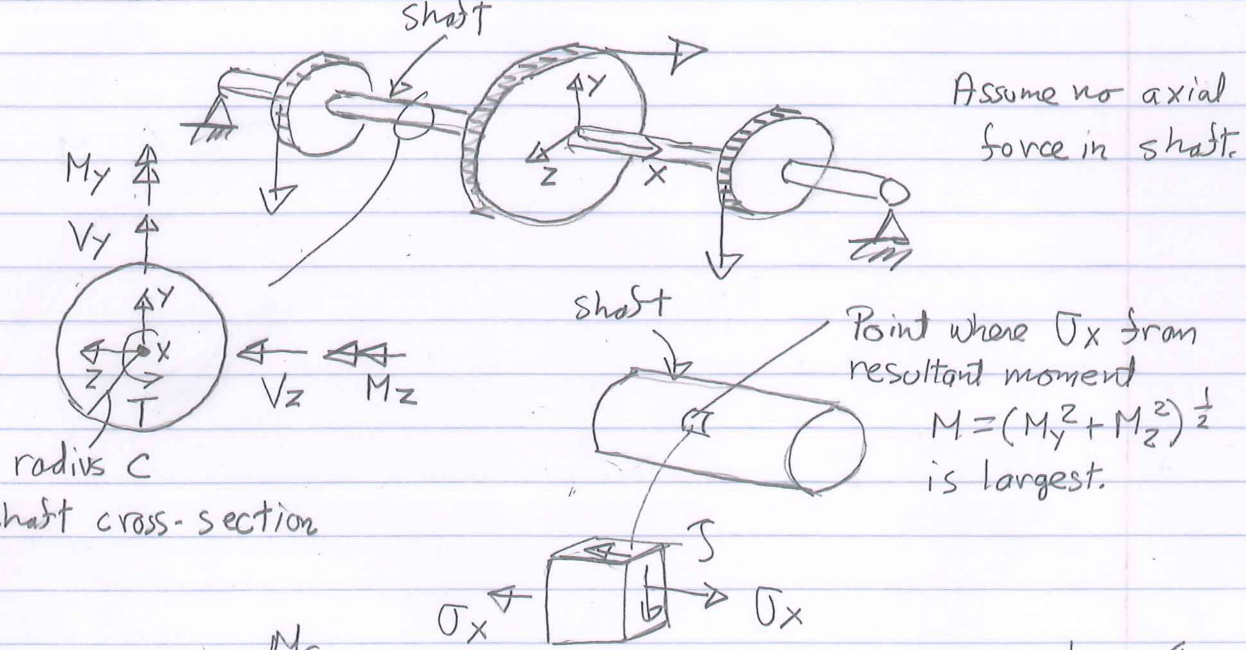
29. Stresses from combined loadings

Linearly elastic, homogeneous, isotropic material

Consider as an example a circular, solid shaft subjected to forces and torques. Find the maximum stresses in the shaft.

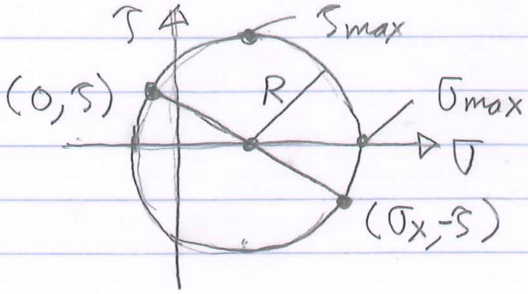


where $\sigma_x = \frac{Mc}{I}$ and $\tau = \frac{Tc}{J}$

$J = \frac{1}{2}\pi c^4$

$I = \frac{1}{4}\pi c^4$

and where the shear stresses from V_y and V_z have been neglected (small).

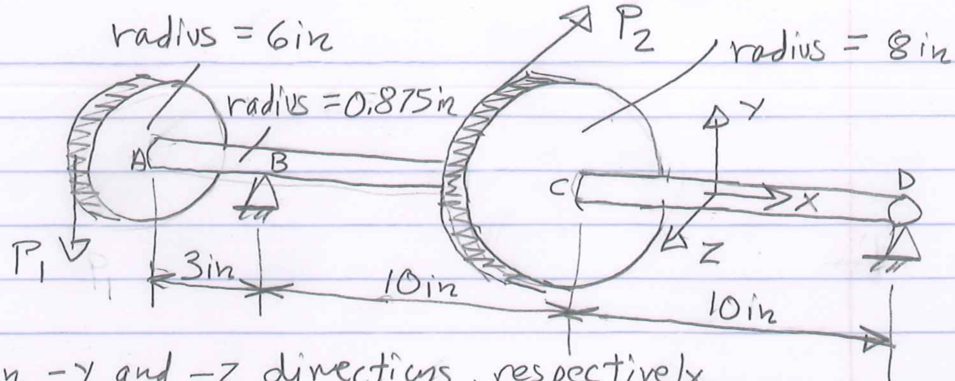


$$R = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau^2 \right]^{\frac{1}{2}} = \frac{2}{\pi c^3} \left(M^2 + T^2 \right)^{\frac{1}{2}}$$

$$\tau_{max} = R, \quad \sigma_{max} = \frac{\sigma_x}{2} + R$$

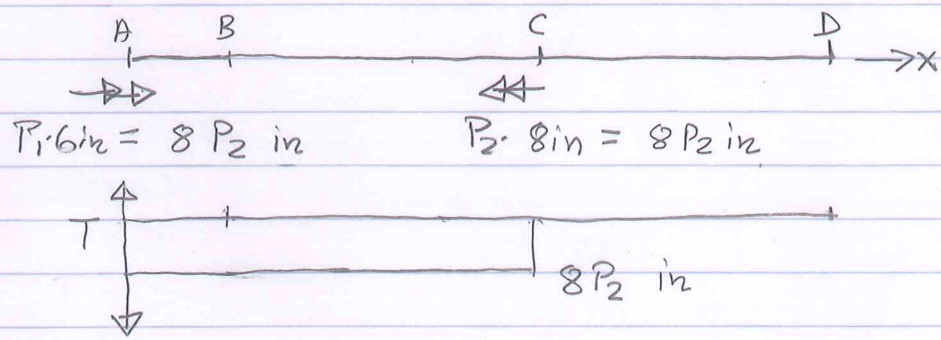
In the above, the point along the shaft is selected so that M and T there make τ_{max} and σ_{max} the largest.

Example

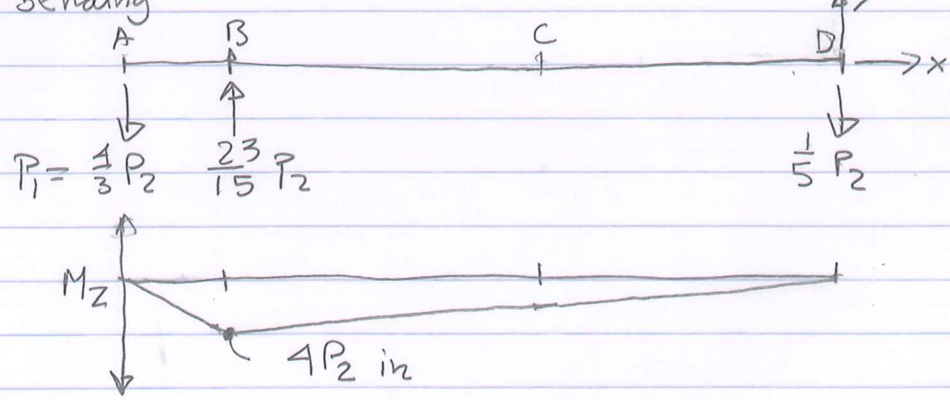


P_1 and P_2 are in $-y$ and $-z$ directions, respectively.
$S_{allow} = 8ksi$
$\sum M$ about axle $= 0 = P_1 \cdot 6in - P_2 \cdot 8in \Rightarrow P_1 = \frac{4}{3} P_2$
Find the maximum value of P_2.

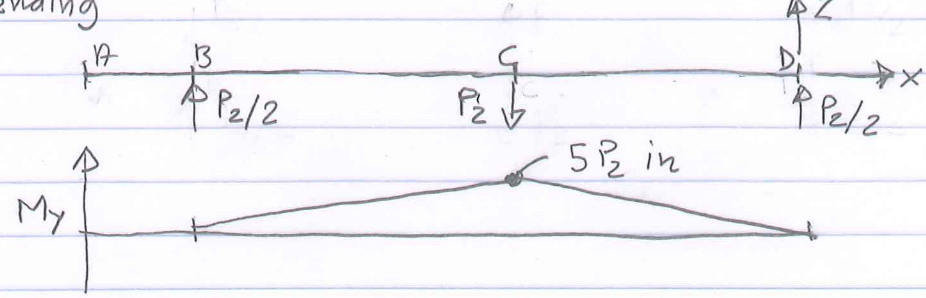
Torque:



Z axis bending



Y axis bending



Maximum stresses will occur at point C (to left of large gear).

$$M^2 = (M_y^2 + M_z^2) = (5 P_2 \text{ in})^2 + (2 P_2 \text{ in})^2 = 29 P_2^2 \text{ in}^2$$

$$T^2 = (8 P_2 \text{ in})^2 = 64 P_2^2 \text{ in}^2$$

$$\tau_{max} = \frac{2}{\pi c^3} (M^2 + T^2)^{\frac{1}{2}} = 9.164 P_2 \frac{1}{\text{in}^2} \qquad (c = 0.875 \text{ in})$$

$$\tau_{max} \leq \tau_{allow}$$

$$9.164 P_2 \frac{1}{\text{in}^2} \leq 8 \text{ ksi}$$

$$P_2 \leq 873 \text{ lb}$$

(shear stress due to P_1 and P_2 acting as shear forces has been omitted)